8. Correlation Analysis

Correlation analysis is the study of relationship between two or more variables. It is the co-variation of two variables.

The measure of correlation is called correlation co-

The weight of a man depends on height. When height increases weight also increases. This is a correlation between weight and height.

The travelling time depends on speed. When the speed is low, the travelling time will be high. When the speed is high, the travelling time will be low. The relationship between time and speed is a correlation.

The correlation is mainly two types, namely positive correlation and negative correlation.

If the two variables move together in the same direction, the correlation is called positive correlation. It is also called direct correlation. In positive correlation, there is an increase in the value of one variable is accompanied by an increase in the value of the other variable; or a decrease in the value of one variable is accompanied by a decrease in the other variable.

Eg. Height and weight - Increase

In negative correlation, the two variables tend to move in opposite directions. It is also called inverse correlation. Here, an increase in the value of one variable is

accompanied by a decrease in the value of the other variable; or a decrease in the value of one variable is accompanied by an increase in the value of the other variable.

Eg. Time and speed.

The correlation is measured by a method called Pearson's coefficient of correlation devised by Karl Pearson. Pearson's coefficient of correlation explains the degree of relationship between two variables.

Pearson's coefficient of correlation is denoted by the symbol y.

The formula for the calculation of Pearson's coeffi-

cient of correlation.

$$\gamma = \frac{\sum dx dy}{\sum dx^2 \times \sum dy^2}$$

 $dx = x - \overline{x} = deviation from \overline{x}$ $dy = y - \overline{y} = deviation from \overline{y}$

 Σ dxdy = Multiply the deviations of x and y and get the total TALL TO THE TOTAL TOTAL

 Σdx^2 = The sum of the square of the deviations of x

 Σdy^2 = The sum of square of the deviations of y

The value of Pearson's coefficient of correlation always lies between +1 and -1.

When γ is +1, then there is perfect positive correlation between the two variables.

When γ is -1, then there is perfect negative correlation between the two variables.

When γ is 0, then there is no relationship between the two variables.

Theoretically, the correlation coefficient values lie inbetween +1 and -1. But actually the values lie between +0.8 and -0.5.

According to **Prof. Bodding** "Wherever some definite connection exists between 2 or more groups, classes or series of data, there is said to be correlation".

A very simple definition is given by A M. Tuttle. "An analysis of the co-variation of two or more variables is usually called correlation.

Uses

- 1. Correlation is very useful to study the relationship between variables.
- 2. Some variables show some kind of relationship; correlation analysis helps in measuring the degree of relationship between the variables.
- 3. The relationship between variables can be verified and tested for significance, with the help of the correlation analysis. The effect of correlation is to reduce the range of uncertainty of our prediction.
- 4. The co-efficient of correlation is a relative measure and we can compare the relationship between variables which are expressed in different units.
 - 5. Sampling error can also be calculated.
- 6. Correlation is the basis for the concept of regression and ratio of variation.

Correlation and Causation

Correlation analysis deals with the association or covariation between two or more variables and helps to determine the degree of relationship between two or more variables. But correlation does not indicate a cause and effect relationship between two variables. It explains only covariation. The high degree of correlation between two variables may exist due to any one or a combination of the following reasons:

1. It is due to pure chance. For example, the relationship between cars production and the children born in a country.

Table 8.1: Degree of correlation.

Degree of Correlation	Positive	Negative
Perfect correlation	+1	- 1
Very high degree of correlation	+0.9 or more	-0.9 or more
Fairly high degree of correlation	from + 0.75 to +0.9	from -0.75 to - 0.9
Moderate degree of correlation	from + 0.5 to +0.75	from - 0.5 to - 0.75
Low degree of correlation	from + 0.25 to +0.5	from - 0.25 to - 0.5
Very low degree of correlation	less than 0.25	less than - 0.25
No correlation	0	0

Types of Correlation

Correlation is classified into many types, but the important ones are:

- 1. Positive and negative
 - 3. Partial and total
- 2. Simple and multiple
- 4. Linear and non-linear

1. Positive and Negative Correlation

Positive and negative correlation depend upon the direction of change of the variables. If two variables tend to move together in the same direction i.e., an increase in the value of one variable is accompanied by an increase in the value of the other variable; or a decrease in the value of one variable is accompanied by a decrease in the value of the other variable; then the correlation is called *positive* correlation or direct correlation. Height and weight, rainfall and yield of crops, age and weight of the baby are examples of positive correlation.

If two variables tend to move together in opposite directions i.e., an increase in the value of one variable is accompanied by a decrease in the value of the other variable; or a decrease in the value of one variable is accompanied by an increase in the value of the other variable; then the correlation is called *negative correlation* or *inverse correlation*. Yield of crops and price are the examples of negative correlation.

2. Simple and Multiple Correlation

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When we study two variables, the relationship is described as simple correlation; Eg. Age and weight of the baby; height and weight of the students, etc. But in a multiple correlation we study more than two variables simultaneously. Eg. Diet, age and weight of the baby.

3. Partial and Total Correlation

The study of two variables excluding some other variables is called *partial correlation*. For example, we study age and weight of the baby eliminating the diet.

In total correlation, all the facts are taken into account.

4. Linear and Non-linear Correlation

If the ratio of change between two variables is uniform, then there will be linear correlation between them. Consider the following:

The ratio of change between the variables is the same

[X:
$$5+5=10$$
; $10+5=15$; $15+5=20$; $20+5=25$;
Y: $3+3=6$; $6+3=9$; $9+3=12$; $12+3=15$]

If we plot these on a graph, we get a straight line like this

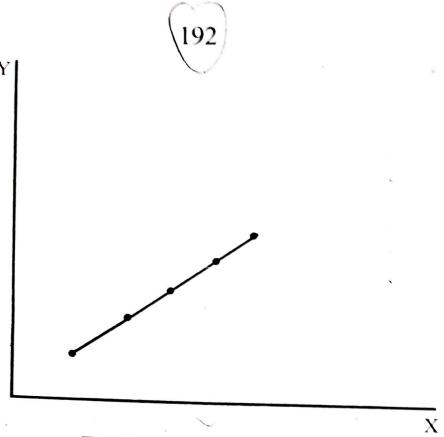


Fig.8.1: Linear correlation.

In a non - linear (curvilinear) correlation, the amount of change in one variable does not bear a constant ratio to the amount of change in the other variables. Consider the following:

The ratio of change between the variable - X is the same [X:5+5=10;10+5=15;15+5=20;20+5=25]

But the ratio of change between the variable-Y is not the same [Y: 3+2=5; 5+1=6; 6+3=9; 9+1=10]

If we plot these on a graph, we get a curve like this

CORRELATION

Kand-Peanson's convertation coefficient,
$$\gamma = \frac{\cos(\alpha, y)}{\sqrt{\alpha}} = \frac{\cos($$

1. Find correlation coefficient from the following data:

X	10	12	18	24	23	27
4	13	18	12	25	30	10

Soln:

Karl-Pearson's correlation coefficient,

α	4	α2	42	24
10	13	(00)	169	130
12	18	144	324	216
18	12	324	144	216
24	25	576	125	600
23	30	529	900	690
27	10	729	100	270
114	108	2402	2262	2122

-0.8698

$$\Sigma x = 114$$

 $\Sigma y = 108$
 $\Sigma x^2 = 2402$
 $\Sigma y^2 = 2262$
 $\Sigma xy = 2122$
 $n = 6$

$$cov(\alpha, 4) = \frac{\sum a_4}{n} - (a)(4)$$

$$= \frac{2122}{6} - (19)(18)$$

$$= 353.6667 - 342$$

OV(X. II) THE LELY

$$0_{\overline{x}} = \sqrt{\frac{2402}{6} - (19)^2} = \sqrt{400.3335 - 361}$$

2 Find peausons co-efficient of covalation

α	10	14	15	28	35	48
ч	74	61	50	54	43	36

		-		42	
1	1	4	21	42	24
	10	14	100	\$476	740
	14	61	196	3721	864
	15	50	225	2500	750
	28	5 4	784	2916	15/2
	35	43	1225	1849	1505
	4 8	36	2304	1296	1728
	150	318	4834	17758	7084
ΣΖΞ	-150				
2y =					
,	- 4834				
Zyz	17758				
Exy.	= 7089	1			
•	h= 6		•		
	7= 2	<u>z</u> =.	150 =	25	
	Y = 2	y = =	318 =	·53	
				(\alpha)	
		-347-	n	- (2)	(4)
		=	7089	- (25)(53)
			•		
				- 1325	
			-143.5		

Find Peauson's co-efficient of concellation for
$$n=12$$
, $\Sigma x^2 = 921$, $\Sigma x = 23$, $\Sigma y^2 = 734$.

$$\Sigma y = 28$$
,
 $\Sigma xy = -432$,

Spearman rank covulation coefficient:

$$P = 1 - \frac{b \Sigma di^2}{h(n^2 - 1)}$$

where, d = Rx-Ry

1. Speanman > 3 stank corolelation coefficient for the following data:

Y	5 0	43	45	36	72	65	47	25
y	62	53	51	25	79	43	60	33

X	у	Rx.	Ry	dore-ky	da
52	62	4	2	2	14
63	53	3	4	-1	1
45	51	6	Ę	1	1
36	25	7	. 8	-1	1
72	79	1	1		0
65	4.5	2	b	-4	16
47	60	5	3	2	4,
25	33	8	7	1	27
proper testing of the second					-1

$$P = 1 - \frac{650^{1}}{500^{1}}$$

$$= 1 - \frac{6(23)}{8(8^{2}-1)}$$

$$= 1 - \frac{169}{8(63)}$$

$$= 1 - \frac{169}{504}$$

$$= 1 - 0.3333$$

= 0.6667.

2. Calculate or cent correlation coefficient.

Soln:-

$$P = 1 - \underline{b \Sigma d^2}$$

$$h(n^2 - 1)$$

RX	Ry	d= Rx-Ry	d2
8	7	1	1
7	5	2	4
Ь	4	2	4
3	1	2	4
2	3	-1	1
	2	-1	-1
٥	ь в	4	1
4	8	-4	16
T			32

$$P = 1 - \frac{6 \times d^{2}}{n \ln^{2} - 1}$$

$$= 1 - \frac{6 (32)}{8(8^{2} - 1)}$$

$$= 1 - \frac{192}{8(63)}$$

$$= 1 - \frac{192}{504} = 1.03810$$

$$= 0.619$$

$$P_{AB} = 1 - 6 \sum_{b=1}^{2} d_{AB}$$

3.

$$P_{Ac} = 1 - \frac{6 \sum d_{Ac}^2}{h(n^2 - 1)}$$

					4.			
RA	Re	Re	dAB	d _{AB}	dec	de a	dac	dae
ι	3	6	-2	4	-3	9	- 5	25
6	5	4	+	1	1	1	2	4
5	8	9	-3	9	-1	1	-4	16
10	4	8	6	36	-4	16	2	4
3	7	1	-4	16	6	36	2	4
2	10	2	- 8	64	8	64	0	0
4	2	7	2	1.4	1	1	A	1
9	1	10	8	64	- 9	81	-1	1
7	6	5		1	1	1	2	4
8	9	7	-1		2	4	1	1
		1		200		2)1	1	60

$$\begin{array}{c} P_{AB} = 1 - 6 \times d_{AB}^{2} \\ \hline n(n^{2}-1) \\ = 1 - 6(200) \\ \hline 10(100-1) \\ = 1 - 1200 \\ \hline qq0 \\ = 1 - 1.2121 \\ = -0.2121 \\ \hline 10(100-1) \\ = 1 - 4284 \\ \hline qq0 \\ = 1 - 1.2970 \\ = -0.2970 \\ \hline P_{CA} = 1 - 6(60) \\ \hline 10(99) \\ = 1 - 560 \\ \hline qq0 \\ = 1 - 0.3636 \\ = 0.6367 \end{array}$$

PAC is measurest to +1.

The pair of judges A and c has heavest apprecach to common taste in music.

Repeated Rank correlation experience.

Coefficient.

X 68 64 75 50 64 80 75 40 55 64 Y 62 58 68 45 81 60 68 48 50 70

Soln!

$$P = \left[-b \left[\sum d^2 + \frac{m(m^2-1)}{12} + \cdots \right] \right]$$

	7				
X	у	PX	Ry	d=Rx-Ry	d ²
68	62	4	5	-)	
64	58	Ь	8	1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	
64	81	6	1	হ	25
80	60	1	6	- 5	25
7 1 40	48	2.5	3.5	-1	
54	\$8	*lo	97	1	0
4	70	Ь	2	4	16
					72

$$\beta = 1 - 6 \left[\frac{5d^2 + m(m^2 + 1)}{12} + \frac{m(m^2 - 1)}{12} + \frac{m(m^2 - 1)}{12} \right]$$

ning

$$= 1 - 6 \left[72 + \frac{5(9-1)}{12} + \frac{2(4-1)}{12} + \frac{2(4-1)}{12} \right]$$

$$= 1 - \frac{6[72 + 2 + 0.5 + 0.5]}{990}$$

$$= 1 - \frac{6[75]}{990}$$

$$= 1 - \frac{450}{990}$$

$$= 1 - 0.4545$$

$$= 0.5455$$

calculate moment correlation from the following bivariate frequently tables.

XX	1	3	5
-	1	t	4
0	3	7	t.
2	6	2	0

Soln!

9. Regression Analysis

Regression is the measures of the average relationship between two or more variables in terms of the original units of the data.

Literally regression means going back or stepping

back.

(The estimation of regression is called regression analysis. In regression analysis, two variables are involved. One variable is called dependent variable and the other is called independent variable.

The yield of rice and rainfall are related. Yield of rice is a dependent variable and rainfall is an independent

variable.

In regression analysis, the value of dependent variable can be calculated from the value of independent variable.

Thus in regression analysis the value of an unknown variable can be calculated from the value of a known variable.

One variable is represented as X and the other variable is represented as Y.

The graphic representation of regression is called regression line.

There are two regression lines. They are,

Regression line of X on Y.

Regression line of Y on X.

Definitions

1. Regression is the measure of the average relationship between two or more variables in terms of the original units of the data"- Blair.

- 2. "Regression analysis attempts to establish the nature of the relationship between variables, that is, to study the functional relationship between the variables and thereby provide a mechanism for predicting or
- and thereby provide a mechanism for predicting or forecasting". -Ya-Lun-Chow.

 . "It is often more important to find out what the relation actually is, in order to estimate or predict one variable (the dependent variable) and statistical technique appropriate in such a case is called Regression analysis". appropriate in such a case is called Regression analysis". -Walis and Robert.
 - 4. "One of the most frequently used techniques in ecoromics and business research, to find a relation betwen two or more variables that are related casually, is regression analysis". -Taro Yamane.

In the regression analysis, the independent variable is dso known as the "regressor" or "predictor" or "explanator" and the dependent variable is known as "reressed" or "explained" variable.

Types of Regression Analysis

Tere are 3 types of regression analysis. They are:

- a. Simple and multiple
- b. Linear and non-linear
- c. Total and partial.

. Simple and Multiple

The regression analysis confined to the study of only wo variables at a time is termed as simple regression. On he otherhand, the regression analysis for studying more than two variables at a time is known as multiple regression.

Regression!

The sucquestion equation on
$$x \in Y$$
 is,
$$(x-\overline{x}) = \frac{\nabla \overline{\nabla x}}{\nabla \overline{y}} (y-\overline{y})$$
The sucquestion equation for $y \in X$ is a by $x = x \in Y$ and $x \in X$.
$$(y-\overline{y}) = \frac{\nabla \overline{y}}{\nabla x} (x-\overline{x})$$

Find two sugression equations and also estimate y when x = 20.

Seln! -

$$(x-\overline{x}) = 7 \frac{\sigma_{\overline{x}}}{\sigma_{\overline{y}}} (y-\overline{y})$$
The suggestion equation of y on X ,
$$(y-\overline{y}) = 7 \frac{\sigma_{\overline{x}}}{\sigma_{\overline{y}}} (x-\overline{x})$$

$$\sigma_{x} = \frac{\cos(\alpha, y)}{\sigma_{x}}$$

$$\cos(\alpha, y) = \frac{\sum \alpha y}{n} - (\frac{\sum y^{2}}{n})$$

$$\sigma_{x} = \sqrt{\frac{\sum \alpha^{2}}{n}} - (\frac{\sum y^{2}}{n})^{2}$$

$$\sigma_{y} = \sqrt{\frac{\sum y^{2}}{n}} - (\frac{y^{2}}{n})^{2}$$

$$\bar{\alpha} = \frac{\epsilon \alpha}{n}$$
, $\bar{\gamma} = \frac{\epsilon \gamma}{n}$

2	4	22	42	24
10	40	100	1600	400
12	38	144	1444	456
13	43	169	1849	559
12	45	144	2025	540
16	37	256	1369	592
15	43	225	1849	645
78	246	1038	10136	3192

$$\overline{\chi} = \frac{\Sigma \chi}{h} = \frac{78}{6} = 13$$

$$\overline{y} = \frac{\Sigma y}{h} = \frac{24b}{6} = 41.$$

$$0 \overline{\chi} = \sqrt{\frac{\Sigma \chi^2}{h} - (\overline{\chi})^2}$$

$$= \sqrt{\frac{1038}{6} - (13)^2}$$

$$= \sqrt{4}$$

$$= 2$$

$$0 \overline{y} = \sqrt{\frac{\Sigma y^2}{h} - (\overline{y})^2}$$

$$= \sqrt{\frac{10136}{6} - 1681}$$

$$= \sqrt{1689.3333 - 1681}$$

$$= \sqrt{2.3333}.$$

$$= 2.8867.$$

$$\cos(x,y) = \frac{2\pi y}{r} - (\pi)(y)$$

$$= \frac{3192}{6} - (13)(41)$$

$$= 5328641 - 533$$

$$= 748/3338.$$

$$= -1.$$

$$\tan(x)$$

$$= \cos(x,y)$$

$$\cos(x)$$

$$= \cos(x)$$

$$\sin(x)$$

$$\sin(x)$$

$$\cos(x)$$

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100 an mile is 1000) 2 1-0.1732.

The required equation of x on y

$$(x-\overline{x}) = 7 \cdot \frac{6x}{6y} (y-\overline{y})$$

 $(x-13)=(-0.1732) - \frac{2}{2.8867} (y-4)$

$$(x-13) = -0.1200(4-41)$$

$$x-13 = -0.12004+4.92$$

$$x = -0.12004+4.92+3$$

$$x = -0.12004+17.92$$

$$(y-\overline{y})=7 \cdot \frac{6y}{6x} \cdot (x-\overline{x})$$

$$(y-41)=1-6 \cdot 1782) \cdot \frac{2 \cdot 8761}{2} \cdot (x-13)$$

$$(y-41)=-6 \cdot 2500x+3 \cdot 25$$

$$y-41=-6 \cdot 2500x+3 \cdot 25+41$$

$$y=-6 \cdot 2500x+3 \cdot 25+41$$

$$y=-6 \cdot 2500x+44 \cdot 25$$

$$y=-6 \cdot 2500(20)+44 \cdot 25$$

$$y=-5+44 \cdot 25$$

$$y=39 \cdot 25$$

9x-3y=16\(\frac{1}{2}\) and \(\frac{1}{2}\)x-4y=40 are the suggestion equations of x (expenditure on advertisement in Rs. Lakes). and y(sales in \(\frac{1}{2}\). Variance of x=2\(\frac{1}{2}\). Find(\(\frac{1}{2}\)\) and \(\frac{1}{2}\) and \(\frac{1}{2}\) and \(\frac{1}{2}\) and \(\frac{1}{2}\) and \(\frac{1}{2}\) \(\frac{1}{2}\) in \(\frac{1}{2}\). Variance of x=2\(\frac{1}{2}\). Find(\(\frac{1}{2}\)\) and \(\frac{1}{2}\) and \(\frac{1}{2}\) in \(\frac{1}\) in \(\frac{1}{2}\) in \(\frac{1}{2}\) in \(\frac{1}{2}\) in \(

Sub
$$\sqrt{5} = 5$$
 in equ(1)
 $9\sqrt{3} - 3(5) = 165$
 $9\sqrt{3} = 165 + 15$
 $9\sqrt{3} = 180$
 $\sqrt{3} = 180$
 $\sqrt{3} = 20$

the sugression equation xony

The degression equation youx

$$y = \frac{73}{44} \times \frac{40}{4}$$

$$= \pm \sqrt{\frac{3}{9}} \times \frac{3}{4}$$

(ii)
$$V(\alpha) = 25$$
. (i.e) $02^{2} = 25$

$$02 = \boxed{25} = 5$$

$$02 = \boxed{25} = 5$$

$$02 = \boxed{25} = 5$$

$$0.3333 = \boxed{0.5} = 5$$

$$0.33333$$

$$0.33333$$

in x=24 lakhs.

$$Y = \frac{3}{4}(2) - \frac{40}{4}$$

$$= \frac{3}{4}(24) - 10$$
 $Y = 8. \text{ Laxins.}$

From the following data. Find the sum of the squares × y
of deviations from 8250 724
the existmetic
mean

Sum of the product 2350 of deviation of x and Y from their respective mean

No. of paints of 10 value

Soun !-

sum of the squares of deviations from the arithmetic mean

$$Z(X-\overline{X})^{2}$$

$$\overline{x}=0$$

$$\Sigma x^{2}=8250$$

$$\Sigma (Y-\overline{Y})^{2} \overline{Y}=0$$

$$\Sigma y^{2}=724$$

$$\gamma = \sum_{x \in \mathbb{Z}} x$$
 (For sum of deviations.).

$$= 2350$$
 $\sqrt{8250}\sqrt{724}$